



A **transformation** is the mapping, or movement, of all the points of a figure in a plane according to a **common operation**.

-A **rigid motion** is a transformation of points in space which **preserves distances**.

-The original figure, before transforming, is called the **pre-image** and the new figure, after transforming, is called the **image**.

A **translation** is a rigid movement that “slides” each point of a figure the same distance and direction.

A **rotation** is a rigid motion that turns a figure about a fixed point.

A **reflection** is a rigid motion that “flips” a figure over a given line.

A **dilation** is a transformation that produces an image that is the same shape as the original, but is a different size. A **dilation** stretches or shrinks the original figure.

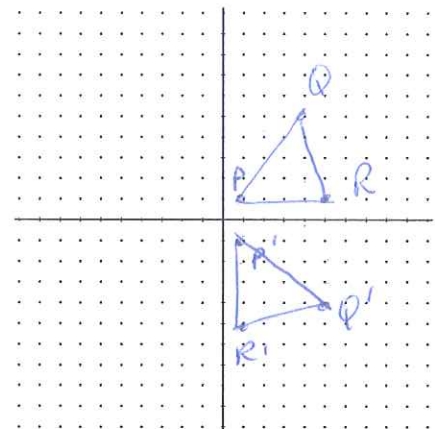
Example 1: Triangle PQR has vertices $P(1, 1)$, $Q(4, 5)$, and $R(5, 1)$. Graph ΔPQR and its image after a rotation of 90° clockwise about the origin.

$P(1, 1)$

$Q(4, 5)$

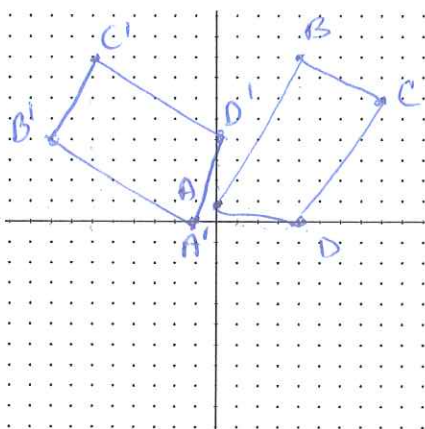
$R(1, 5)$

90° cw
 $(x, y) \rightarrow (y, -x)$



Example 2: Graph the following figures and their images. Compare the coordinates of the image and preimage. Quadrilateral $ABCD$, with points $A(0, 1)$, $B(4, 8)$, $C(8, 6)$, and $D(4, 0)$.

a. 90° counterclockwise about the origin



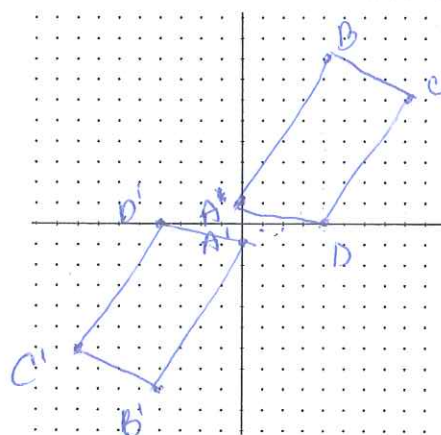
$A'(-1, 0)$

$B'(-8, 4)$

$C'(-6, 8)$

$D'(0, 4)$

b. 180° counterclockwise about the origin



$A'(0, -1)$

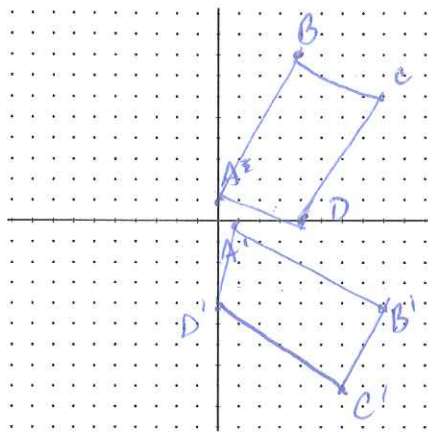
$B'(-4, -8)$

$C'(-8, -6)$

$D'(-4, 0)$

$90^\circ \text{ ccw } (x, y) \rightarrow (-y, x)$

c. 270° counterclockwise about the origin



$A'(1, 0)$
 $B'(8, -4)$
 $C'(6, -8)$
 $D'(0, -4)$

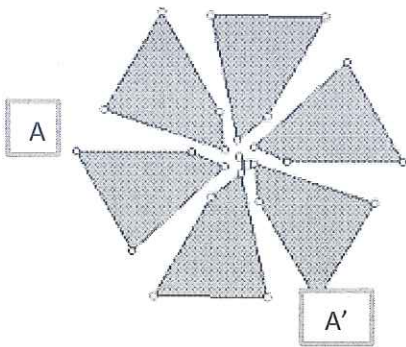
270° CCW $(x, y) \rightarrow (y, -x)$

d. What is the rule for rotating a figure about the origin for each given degree measure below?

90° Rotation: $(x, y) \rightarrow (y, -x)$
 180° Rotation: $(x, y) \rightarrow (-x, -y)$
 270° Rotation: $(x, y) \rightarrow (-y, x)$

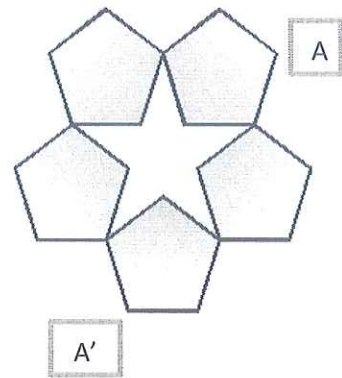
Example 3: For each figure below, answer the following questions.

- Determine the ^{# of} angles of rotational symmetry the figure has.
- Determine the angle of rotation for A to map onto A'.
- Determine all the angles of rotation.



a.

- 5
- 120° CW or 240° CCW
- $60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ$



b.

- 4
- 144° CW or 216° CCW
- $72^\circ, 144^\circ, 216^\circ, 288^\circ$

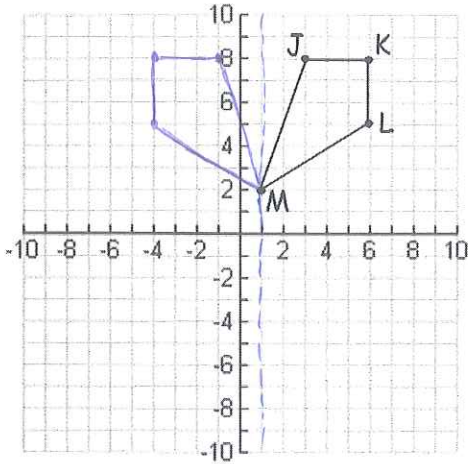
$\frac{72}{\sqrt{360}}$

Example 4: Determine the coordinates of the transformed image after the specified rotation about the origin.

- $\triangle JKL$ has vertices $J(2, 6)$, $K(5, 2)$, and $L(7, 5)$; 90° counterclockwise. 90° CCW $(x, y) \rightarrow (-y, x)$
 $J'(-6, 2)$ $K'(-2, 5)$ $L'(-5, 7)$
- Trapezoid $ABCD$ has vertices $A(-7, -2)$, $B(-6, -6)$, $C(-1, -1)$, and $D(-5, 0)$; 180° $180^\circ (x, y) \rightarrow (-x, -y)$
 $A'(7, 2)$ $B'(6, 6)$ $C'(1, 1)$ $D'(5, 0)$
- Parallelogram $MPQV$ has vertices $M(-6, 3)$, $P(-2, 3)$, $Q(-3, -2)$, and $V(-7, -2)$; 270° clockwise. 270° CW $(x, y) \rightarrow (-y, x)$
 $M'(-3, -6)$ $P'(-3, -2)$ $Q'(2, -3)$ $V'(2, -7)$

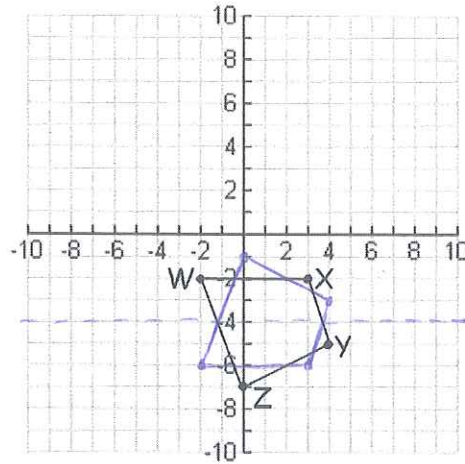
Example 5: Graph each figure and its reflection about the given line. Determine the coordinates of the transformed image.

a) $JKLM$; $x = 1$



$J'(-4, 8)$ $K'(-9, 8)$
 $L'(-4, 5)$ $M'(1, 2)$

b) $WXYZ$; $y = -4$



$W'(-2, -6)$ $X'(3, -6)$
 $Y'(4, -3)$ $Z'(0, -1)$

Example 6: Determine the coordinate of the image after the given reflection.

a) Square $JKLM$ with vertices $J(-5, 6)$, $K(0, 6)$, $L(0, 2)$, and $M(-3, 2)$ in the y -axis.

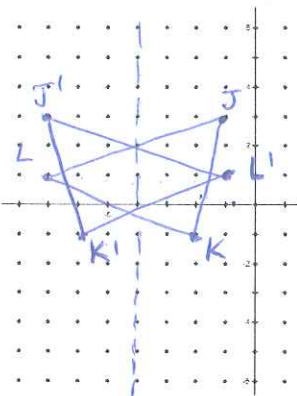
$J'(5, 6)$ $K'(0, 6)$ $L'(0, 2)$ $M'(3, 2)$

b) Triangle FGH with vertices $F(-4, 2)$, $G(-3, -1)$, and $H(-5, -1)$ in the line $y = 2$.

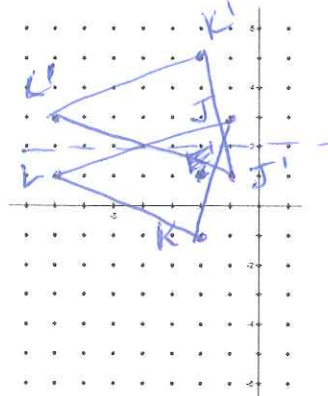
$F'(-4, 2)$ $G'(-3, 5)$ $H'(-5, 5)$

Example 7: Triangle JKL has vertices $J(-1, 3)$, $K(-2, -1)$, and $L(-7, 1)$. Graph the triangle and its image in the given line.

a) $x = -4$



b) $y = 2$



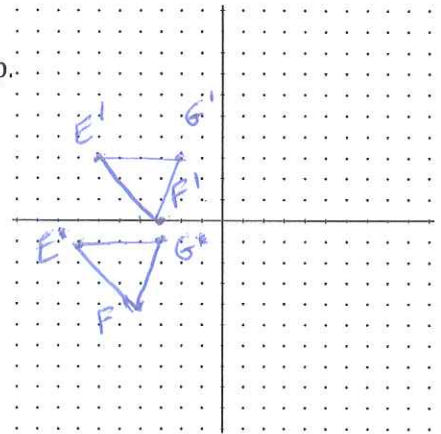
Example 8: Graph the figure and its image. $\triangle EFG$ with vertices $E(-7, -1)$, $F(-4, -4)$, and $G(-3, -1)$ translated 1 units right and 4 units up.

$$E'(-6, 3)$$

$$F'(-3, 0)$$

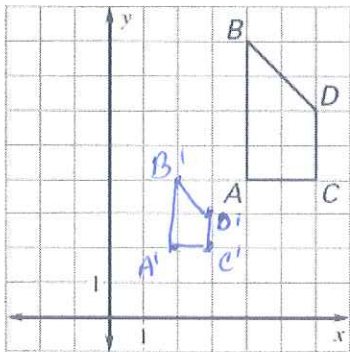
$$G'(-2, 3)$$

$$(x+1, y+4)$$



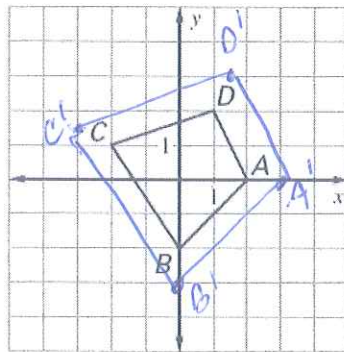
Example 9: Draw a dilation of the figure using the given scale factor and give the prime coordinates.

3. $k = \frac{1}{2}$



$$A'(1, 1) \quad B'(1, 2) \quad C'(1.5, 1) \quad D'(1.5, 1.5)$$

4. $k = 1\frac{1}{2}$



$$A(2, 0)$$

$$B(0, -2)$$

$$C(-2, 1)$$

$$D(1, 2)$$

$$A'(3, 0)$$

$$B'(0, -3)$$

$$C'(-3, 1.5)$$

$$D'(1.5, 3)$$

Example 10: Determine the coordinates of the image along the given translation.

a) $\triangle ABC$ with vertices $A(1, 6)$, $B(3, 2)$, and $C(4, 7)$ by the translation $(x + 4, y - 1)$.

$$A'(5, 5) \quad B'(7, 1) \quad C'(8, 6)$$

b) Quadrilateral with vertices $F(-4, -2)$, $G(-1, 1)$, $H(0, -4)$, and $J(-3, -6)$ translated left 5 units and down 2 units.

$$F'(1, -4) \quad G'(4, -1) \quad H'(5, -6) \quad J'(2, -8)$$

c) Trapezoid with vertices $J(-4, -2)$, $K(-1, -2)$, $L(0, -5)$, and $M(-5, -5)$ by the translation $\langle 6, 5 \rangle$.

$$J'(2, 3) \quad K'(5, 3) \quad L'(6, 0) \quad M'(1, 0)$$