

Essential Questions: How can I...

- write and solve algebraic and inequalities equations
- solve a literal equation for a specific variable
- write an algebraic proof for a given equation
- write and solve a system of linear equations and inequalities by graphing, substitution and elimination
- solve a linear programming problem
- model situations with linear functions, evaluate functions, characteristics of functions
- write explicit and recursive formulas of arithmetic sequences

Solve each equation.

1. $30 + v = 8v - 2(-3 + 5v)$
 $30 + v = 8v + 6 - 10v$
 $-6 - v \quad -v - 6$
 $\frac{24}{-3} = \frac{-3v}{-3}$
 $v = -8$

2. $5k - 4k = -(4 - 6k) - 2(5k + 8)$
 $k = -4 + 6k - 10k - 16$
 $-6k + 10k \quad -6k + 10k$
 $4k = -20$
 $k = -4$

3. $-\frac{15}{4}r + \frac{4}{3}r = -\frac{29}{48}$

4. $-\frac{9}{10} - \frac{7}{6}a = \frac{3}{2}a - 2\left(\frac{a+6}{5}\right)$

$\frac{15}{29} \cdot \frac{29}{12} r = \frac{29}{48} \cdot \frac{12}{29}$
 $r = \frac{1}{4}$

$-\frac{9}{10} - \frac{7}{6}a = \frac{3}{2}a - 2a - \frac{12}{5}$
 $+\frac{12}{5} \quad +\frac{12}{5}a \quad +\frac{12}{5}a \quad +\frac{12}{5}$
 $\frac{3}{2} \cdot \frac{3}{2} = \frac{2}{3}a \cdot \frac{1}{2}$
 $a = \frac{9}{4}$

Translate the word problem into an algebraic inequality, and then solve to answer the question.

5. The sum of a number and 81 is greater than the product of -3 and that number. What are the possible values for the number?

$x + 81 > -3x$
 $81 > -4x$
 $-\frac{81}{4} < x$
 OR $-20.25 < x$

6. ATT offers you two plans for cell service. Plan A allows you to pay \$20 a month plus \$.05 for each text message or plan B is \$40 per month with unlimited texting. For what number of text messages each month will option A be the cheaper plan?

$20 + .05x < 40$
 $-.05x < \frac{20}{.05}$
 $x < 400$

7. Adrian works in New York City and makes \$42 per hour. She works in an office and must get her suit dry cleaned everyday for \$35. If she wants to make more than \$260 a day, at least how many hours must she work?

$42x - 35 > 260$
 $42x > 295$
 $x > 7.02$

Solve the following equations for the variable specified.

8. $Q = 3a + 5ac$, for a
 $Q = a(3 + 5c)$
 $\frac{Q}{3 + 5c} = a$

9. $y = \frac{3}{5}\left(\frac{1}{2}x - 32\right)$, for x
 $\frac{9}{5}y = \frac{1}{2}x - 32$
 $\frac{9}{5}y + 32 = \frac{1}{2}x$
 $2\left(\frac{9}{5}y + 32\right) = x$ OR $\frac{18}{5}y + 64 = x$

10. $S = 2lh + 2lw + 2hw$, for w
 $S - 2lh = 2w(l + h)$
 $\frac{S - 2lh}{2(l + h)} = w$

11. $I = \frac{nE}{nr + R}$, for n

$Inr + IR = nE$
 $IR = nE - Inr$
 $IR = n(E - Ir)$
 $\frac{IR}{E - Ir} = n$

List the property used in each step.

13. $\frac{5b+4}{3} = b - 6$, for b

12. $\frac{3}{a-b} = \frac{c}{2a}$, for a

$6a = ac - bc$
 $6a - ac = -bc$
 $a(6 - c) = -bc$
 $a = \frac{-bc}{6 - c}$

14. $4ab + 1 = 6a - 3$, for a

$4ab - 6a + 1 = 6a - 6a - 3$ Subtraction
 $4ab - 6a + 1 = 0 - 3$ Simplify (combined like terms)
 $4ab - 6a + 1 = -3$ Identity (Additive)
 $4ab - 6a + 1 - 1 = -3 - 1$ Subtraction
 $4ab - 6a + 0 = 4$ Simplify (combined like terms)
 $4ab - 6a = 4$ Identity (Additive)
 $a(4b - 6) = 4$ Factor GCF
 $\frac{a(4b - 6)}{(4b - 6)} = \frac{4}{(4b - 6)}$ Division
 $a = \frac{4}{4b - 6}$ Identity (Multiplicative)

$3 \cdot \frac{5b+4}{3} = 3 \cdot (b - 6)$ Multiplication
 $5b + 4 = 3 \cdot (b - 6)$ Identity (Multiplicative)
 $5b + 4 = 3b - 18$ Distributive
 $5b - 3b + 4 = 3b - 3b - 18$ Subtraction
 $2b + 4 = 0 - 18$ Simplify (combined like terms)
 $2b + 4 = -18$ Identity (Additive)
 $2b + 4 - 4 = -18 - 4$ Subtraction
 $2b + 0 = -22$ Simplify (combined like terms)
 $2b = -22$ Identity (Additive)
 $\frac{2b}{2} = \frac{-22}{2}$ Division
 $b = -11$ Simplify

Solve each system by substitution. 15. 16.

$2x + 6y = 22$
 $x + 3y = 11 \rightarrow x = -3y + 11$
 $2(-3y + 11) + 6y = 22$
 $-6y + 22 + 6y = 22$
 $22 - 22 = 0$
 Infinite Solutions

Solve each system by elimination.

$9x - 4y = 4$
 $-18x + 9y = -18$
 $\pm 18x - 8y = 8$
 $y = -10$

$9x - 4(-10) = 4$
 $9x + 40 = 4$
 $9x = -36$
 $x = -4$

Set up a system of equations for each situation and solve the system using substitution or elimination.

17. Molly's Custom Kitchen Supplies sells handmade forks and spoons. It costs the store \$1.70 to buy the supplies to make a fork and \$1.30 to buy the supplies to make a spoon. The store sells forks for \$5.60 and spoons for \$5.40. Last April Molly's Custom Kitchen Supplies spent \$37.90 on materials for forks and spoons. They sold the finished products for a total of \$147.20. How many forks and how many spoons did they make last April?

$560(1.70x + 1.30y = 37.90)$
 $-170(560x + 540y = 147.20)$
 $952x + 728y = 21224$
 $-952x - 918y = -25024$
 $-190y = 3800$
 $y = 20$ spoons
 $x = 7$ forks

18. All 231 students in the Math Club went on a field trip. Some students rode in vans which hold 7 students each and some students rode in buses which hold 25 students each. How many of each type of vehicle did they use if there were 15 vehicles total?

$$\begin{aligned} 7x + 25y &= 231 \\ x + y &= 15 \\ \rightarrow x &= -y + 15 \end{aligned}$$

$$\begin{aligned} 7(-y + 15) + 25y &= 231 \\ 18y + 105 &= 231 \\ 18y &= 126 \\ y &= 7 \quad x = 8 \end{aligned}$$

8 vans
7 buses

19. Matt and Ming are selling fruit for a school fundraiser. Customers can buy small boxes of oranges and large boxes of oranges. Matt sold 3 small boxes of oranges and 14 large boxes of oranges for a total of \$203. Ming sold 11 small boxes of oranges and 11 large boxes of oranges for a total of \$220. Find the cost each of one small box of oranges and one large box of oranges.

Small box = \$7
Large box = \$13

$$\begin{aligned} 11(3x + 14y &= 203) \\ -3(11x + 11y &= 220) \end{aligned}$$

$$\begin{aligned} 33x + 154y &= 2233 \\ -33x - 33y &= -660 \end{aligned}$$

$$121y = 1573 \quad y = 13$$

Use linear programming to find the maximum or minimum of the following system of inequalities.

20. You are taking a test in which items of type A is worth 10 points and items of type B are worth 15 points. It takes 3 minutes to answer each item of type A and 6 minutes to answer each item of type B. The total time allowed is 60 minutes, and you may not answer more than 16 questions. Assuming all of your answers are correct, how many items of each type should you answer to get the highest score?

Objective: $S = 10x + 15y$

Constraints: $x + y \leq 16 \rightarrow y \leq -x + 16$

$3x + 6y \leq 60 \rightarrow y \leq -\frac{1}{2}x + 10$

$x \geq 0$

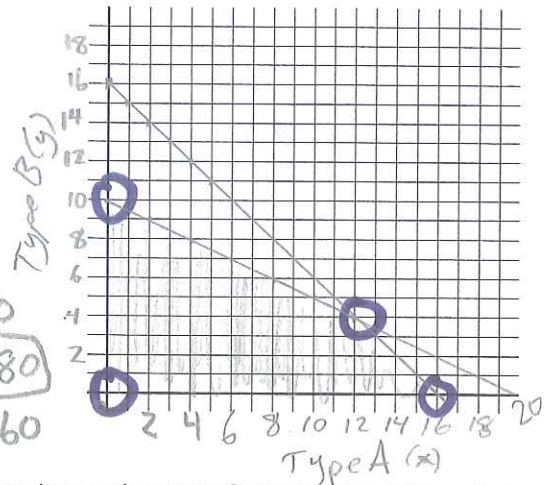
$y \geq 0$

Vertices

$(0, 10) \quad S = 10(0) + 15(10) = 150$

$(12, 4) \quad S = 10(12) + 15(4) = 180$

$(16, 0) \quad S = 10(16) + 15(0) = 160$



You should answer 12 type A and 4 type B.

21. A company can manufacture at least 20 and at most 90 radios per day. It can also manufacture at least 40 and at most 120 TV's in a day. The company must produce a minimum of 80 but no more than 120 items in one day. If their profit is \$50 per radio and \$80 per TV, how many should they manufacture to maximize their profits?

$x = \text{Radios}$

$P = 50x + 80y$

$y = \text{TVs}$

$x + y \geq 80$ OR $80 \leq x + y \leq 120$

$x + y \leq 120$

$20 \leq x \leq 90$

$40 \leq y \leq 120$

Difficult to graph

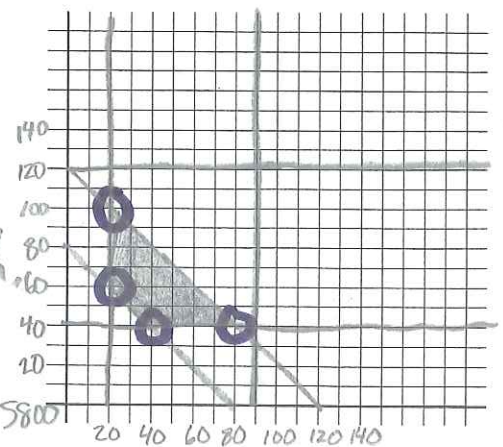
Vertices

$(20, 60) \quad P = 50(20) + 80(60) = 5800$

$(20, 100) \quad P = 50(20) + 80(100) = 9000$

$(40, 40) \quad P = 50(40) + 80(40) = 5200$

$(80, 40) \quad P = 50(80) + 80(40) = 7200$



The company should make 20 radios and 100 TVs to maximize their profits.

Write a function using function notation for each of the following situations.

22. A backyard pool contains 500 gallons of water. It is filled with additional water at a rate of 6 gallons per minute.

$$V(t) = 500 + 6t$$

23. A helicopter flying at 3505 feet begins its descent. It descends at a rate of 470 feet per minute.

$$H(t) = 3505 - 470t$$

For each of the following functions, find the indicated values.

24. Let $f(x) = -4x + 12$ and $g(x) = x^2 - 5x + 2$. Find the following values.

a. $f(10)$

$$f(10) = -4(10) + 12 = -28$$

b. $g(-2)$

$$g(-2) = (-2)^2 - 5(-2) + 2 = 16$$

c. $2f(-1) - 3g(4)$

$$2(16) - 3(-2) = 38$$

25. $f(x) = 240x + 1200$

a. $f(-20) = -3600$

b. Find x when $f(x) = 720$

$$720 = 240x + 1200$$

$$-480 = 240x \quad x = -2$$

Write a function for the situation and answer the questions.

26. A hot air balloon at 4000 feet begins its descent. It descends at a rate of 200 feet per minute.

a. Identify the independent and dependent quantities.

time \downarrow height

b. Write a function to show the balloon's elevation as a function of time in minutes.

$$h(t) = 4000 - 200t$$

c. Graph the function.

d. What is the domain of the function? $[0, 20]$

e. What is the range of the function? $[0, 4000]$

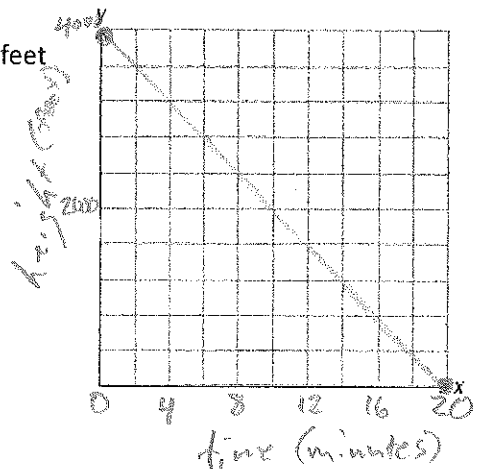
f. Should the points be connected? (Is the graph discrete or continuous?) Yes

g. For what times is the balloon at least 2500 feet high?

$$2500 \leq 4000 - 200t \quad 7.5 \geq t$$

h. After what time is the balloon's height less than 1000 feet?

$$1000 \geq 4000 - 200t \quad 15 \leq t$$



27. Given the following arithmetic sequence, find the recursive formula and the explicit formula for 13, 6, -1, -8, ...

$$a_n = 13 - 7(n-1) \quad a_n = a_{n-1} - 7$$

28. The eleventh term in an arithmetic sequence is 34. If the common difference is -5, what is the simplified explicit formula for the sequence?

$$34 = a_1 - 5(11-1)$$

$$34 = a_1 - 50$$

$$84 = a_1$$

$$a_n = 84 - 5(n-1)$$

$$a_n = 84 - 5n + 5$$

$$a_n = 89 - 5n$$

29. If the tenth term in an arithmetic sequence is 55 and the fourteenth term is 83, what is the simplified explicit formula for the sequence?

$$d = \frac{83 - 55}{14 - 10} = \frac{28}{4} = 7$$

$$55 = a_1 + 7(10-1) \quad -8 = a_1$$

$$a_n = -8 + 7(n-1) \quad a_n = -15 + 7n$$

30. A recovering heart attack patient is told to get on a regular walking program. The patient is told to walk a distance of 5 km the first week, 8 km the second week, 11 km the third week, and so on for a period of 10 weeks. Find the distance the patient will walk for the first 10 weeks. If the patient continues to increase walking, how far will the patient walk in the 20th week?

5, 8, 11, ...

$$a_n = 5 + 3(n-1)$$

$$a_{20} = 5 + 3(20-1)$$

$$a_{20} = 62$$