Acc Alg I/Geo A Unit 4 Test Review Name Key

| | |
|---|---|
| - Properties of Exponents | - Solving Exponential Equations |
| -Graphing Exponential Functions | -Modeling with Exponential Functions |
| -Geometric Sequences | -Exponential Growth, Decay and Compound Interest |

**Use the properties of exponents to simplify the following expressions.**

1) $(2b^3)^4 \cdot 2a^4b^0$ $= 16b^{12} \cdot 2a^4$ $= \boxed{32a^4b^{12}}$

2) $(m^{-1}n^3)^{-4} \cdot 2m^4$ $= m^4n^{-12} \cdot 2m^4$ $= \dfrac{m^4 \cdot 2m^4}{n^{12}} = \boxed{\dfrac{2m^8}{n^{12}}}$

3) $\dfrac{3a^4b^{-4}}{a^{-1}b^2 \cdot 2a^2}$ $= \dfrac{3a^4a^1}{b^4 \cdot b^2 \cdot 2a^2} = \boxed{\dfrac{3a^3}{2b^6}}$

4) $\dfrac{4nm^3 \cdot 4m^2n^{-2}}{4m^{-3}n^4}$ $= \dfrac{16n^{-1}m^5}{4m^{-3}n^4} = \dfrac{16m^5m^3}{4n^1n^4} = \boxed{4m^8/n^5}$

**Solve each exponential equation.** Remember to check these!

11) $625^{3-2x} = \dfrac{1}{25}$

$5^{4(3-2x)} = 5^{-2}$

$5^{12-8x} = 5^{-2}$

$12 - 8x = -2$

$-8x = -14$

$\boxed{x = 7/4}$

12) $8^{3-2x} = 16$

$2^{3(3-2x)} = 2^4$

$2^{9-6x} = 2^4$

$9 - 6x = 4$

$-6x = -5$

$\boxed{x = \dfrac{5}{6}}$

13) $9^{-3m-2} = \left(\dfrac{1}{81}\right)^{2m}$

$3^{2(-3m-2)} = 3^{-4(2m)}$

$3^{-6m+4} = 3^{-8m}$

$-6m + 4 = -8m$

$2m = -4$

$\boxed{m = -2}$

14) $32^{-v+1} = 64^{3v+3}$

$2^{5(-v+1)} = 2^{6(3v+3)}$

$2^{-5v+5} = 2^{18v+18}$

$-5v + 5 = 18v + 18$

$-13 = 23v$

$\boxed{v = \dfrac{-13}{23}}$

15) $36^{-2x} \cdot 216^{-3x} = \dfrac{1}{216}$

$6^{2(-2x)} \cdot 6^{3(-3x)} = 6^{-3}$

$6^{-4x-9x} = 6^{-3}$

$6^{-13x} = 6^{-3}$

$-13x = -3$

$\boxed{x = \dfrac{3}{13}}$

16) $36^{3b+1} \cdot \left(\dfrac{1}{216}\right)^{1-3b} = 36^{2b-1}$

$6^{2(3b+1)} \cdot 6^{-3(1-3b)} = 6^{2(2b-1)}$

$6^{6b+2-3+9b} = 6^{4b-2}$

$15b - 1 = 4b - 2$

$11b = -1$

$\boxed{b = -\dfrac{1}{11}}$

17) $\dfrac{243}{27^{-2p}} = 81^p$

$\dfrac{3^5}{3^{3(-2p)}} = 3^{4p}$

$3^{5-(-6p)} = 3^{4p}$

$3^{5+6p} = 3^{4p}$

$5 + 6p = 4p$

$2p = -5$

$\boxed{p = \dfrac{-5}{2}}$

18) $\dfrac{16^{2r}}{64^{1-2r}} = 64^{r+1}$

$\dfrac{4^{4r}}{4^{3-6r}} = 4^{3r+3}$

$4^{10r-3} = 4^{3r+3}$

$10r - 3 = 3r + 3$

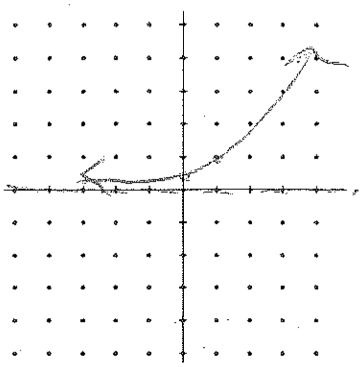
$7r = 6$

$\boxed{r = \dfrac{6}{7}}$

**Graph the following Exponential Functions. Fill in the chart with the characteristics.**

19) $f(x) = \frac{1}{3}3^x$

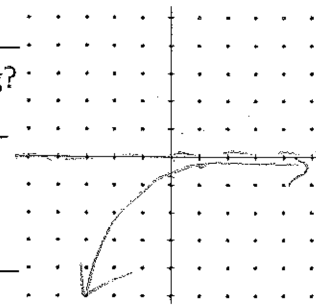
CP: $(0, \frac{1}{3})(1, 1)$



y – Intercept $(0, \frac{1}{3})$
Increasing or Decreasing?
Asymptote? $y = 0$
Domain $(-\infty, \infty)$
Range $(0, \infty)$
End Behavior
As $x \to \infty, y \to \infty$
As $x \to -\infty, y \to 0$

20) $f(x) = -\frac{1}{2}\left(\frac{1}{2}\right)^x$

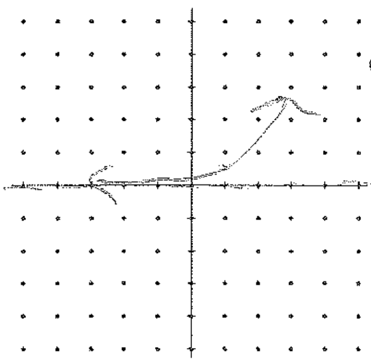
$(1, -\frac{1}{4})$
CP: $(0, -\frac{1}{2})$



y – Intercept $(0, -\frac{1}{2})$
Increasing or Decreasing?
Asymptote? $y = 0$
Domain $(-\infty, \infty)$
Range $(-\infty, 0)$
End Behavior
$x \to \infty, y \to 0$
$x \to -\infty, y \to -\infty$

21) $f(x) = \frac{1}{4}2^x$

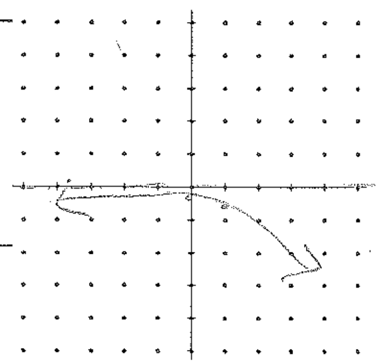
CP: $(0, \frac{1}{4})(1, \frac{1}{2})$



y – Intercept $(0, \frac{1}{4})$
Increasing or Decreasing?
Asymptote? $y = 0$
Domain $(-\infty, \infty)$
Range $(0, \infty)$
End Behavior
$x \to \infty, y \to \infty$
$x \to -\infty, y \to 0$

22) $f(x) = -\frac{1}{4}2^x$

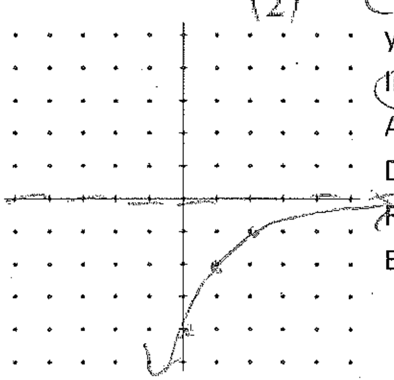
CP: $(0, -\frac{1}{4})(1, -\frac{1}{2})$



y – Intercept $(0, -\frac{1}{4})$
Increasing or Decreasing?
Asymptote? $y = 0$
Domain $(-\infty, \infty)$
Range $(0, -\infty)$
End Behavior
$x \to \infty, y \to -\infty$
$x \to -\infty, y \to 0$

23) $f(x) = -2 \cdot \left(\frac{1}{2}\right)^{x-1}$

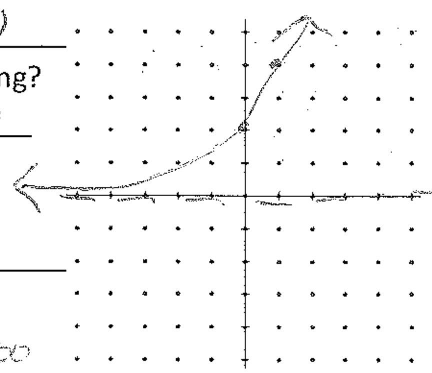
CP: $(1, -2)(2, -1)$



y – Intercept $(0, -4)$
Increasing or Decreasing?
Asymptote? $y = 0$
Domain $(-\infty, \infty)$
Range $(-\infty, 0)$
End Behavior
$x \to \infty, y \to 0$
$x \to -\infty, y \to -\infty$

24) $f(x) = 4 \cdot 2^{x-1}$

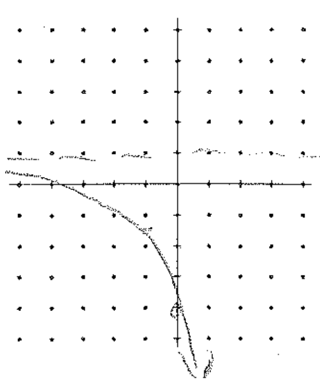
CP: $(1, 4)(2, 8)$



y – Intercept $(0, 2)$
Increasing or Decreasing?
Asymptote? $y = 0$
Domain $(-\infty, \infty)$
Range $(0, \infty)$
End Behavior
$x \to \infty, y \to \infty$
$x \to -\infty, y \to 0$

25) $f(x) = -5 \cdot 2^x + 1$

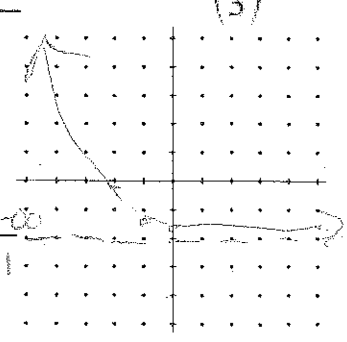
CP: $(0, -4)(1, -9)$



y – Intercept $(0, -4)$
Increasing or Decreasing?
Asymptote? $y = 1$
Domain $(-\infty, \infty)$
Range $(-\infty, 1)$
End Behavior $x \to \infty, y \to -\infty$
$x \to -\infty, y \to 1$
$(-1, -1.5)$

26 $y = 2 \cdot \left(\frac{1}{3}\right)^{x+2} - 2$

CP: $(-2, 0)(-1, -\frac{4}{3})$



y – Intercept $(0, -1.8)$
Increasing or Decreasing?
Asymptote? $y = -2$
Domain $(-\infty, \infty)$
Range $(-2, \infty)$
End Behavior
$x \to \infty, y \to -2$
$x \to -\infty, y \to \infty$

**Set up an exponential model for each situation. Use the model to answer the questions.**

27) A computer depreciates 25% every six months. If the computer is valued at $3000 today, how much will it be worth in 2 years?

$P_t = 3000(1-.25)^t$ $t =$ every 6 months

$P_4 = 3000(.75)^4 = \$949.22$

28) The value of land in a certain town increases by 6.75% each year. If you bought a parcel of this land in 1995 for $150, 000, what was the land worth in 2003?

$P_t = 150{,}000(1+.0675)^t$ $t =$ yearly

$P_8 = 150{,}000(1.0675)^8 = \$252{,}949.79$

29) The population in a city in 1990 was 213,426. The population increased at a rate of 3.1% each year. What was the approximate population in the city in 2000?

$P_t = 213426(1+.031)^t$ $t =$ yearly

$P_{10} = 213426(1.031)^{10} = 289{,}623$ people

30) You deposited $11, 000 into a savings account that compounds quarterly. The account has an interest rate of 3.7%. What would be the current principle after 3 years and 6 months?

$P_t = 11{,}000\left(1+\frac{.037}{4}\right)^{4t}$

$P_{3.5} = \$12{,}513.40$

31) An investment of $1248 earns 10.9% interest. Compare the value of the investment after 5 years, 10 years, and 20 years between simple interest and interest compounded semi-annually.

| | simple | compounded semi-annually |
|---|---|---|
| MODEL: | $1248 + (.109 \times 1248)t$ | $1248\left(1+\frac{.109}{2}\right)^{2t}$ |
| 5 yrs. | \$1928.16 | \$2,121.68 |
| 10 yrs. | \$2608.32 | \$3,607.00 |
| 20 yrs. | \$3968.64 | \$10,425.05 |

**Identify whether each of the following sequences are arithmetic or geometric. Identify the common difference/ratio and find the 6th and 15th term of each sequence.**

32) $12, 3, \frac{3}{4}, \frac{3}{16} \ldots$

Model: $a_n = 12\left(\frac{1}{4}\right)^{n-1}$

6th term: $a_6 = \frac{3}{256}$ or $.0117$

15th term: $a_{15} = 4.5 \times 10^{-8}$

33) $4, -24, 144, -864 \ldots$

Model: $a_n = 4(-6)^{n-1}$

6th term: $a_6 = -31{,}104$

15th term: $a_{15} = 3.13 \times 10^{11}$

**Use what you know about sequences to answer the following questions.**

34) The seventh term in a geometric sequence is -3645. If the common ratio is -3, what is the simplified explicit formula for the sequence?

$$-3645 = a_1(-3)^{7-1}$$

$$\frac{-3645}{729} = \frac{a_1 \cancel{729}}{729} \qquad a_1 = -5 \qquad a_n = -5(-3)^{n-1}$$

35) If the fourth term in a geometric sequence is 192 and the ninth term is 196,608, what is the simplified explicit formula for the sequence?

192
4

196,608
9

5

$$\frac{196{,}608}{192} = 1024$$

$$x^5 = 1024 \qquad \underline{x = 4} \qquad \underline{r = 4}$$

$$\frac{192}{64} = \frac{a_1(4)^{4-1}}{64}$$

$$a_1 = 3 \qquad \boxed{a_n = 3(4)^{n-1}}$$

**Write the explicit formula and use it to solve this situation.**

36) Bradley makes two phone calls to his friends to tell them school is cancelled because of snow. Each of those friends makes two calls to tell their friends the same news. Each of those friends makes two calls to tell their friends the same news, and so on. Find the number of calls made for the first 10 terms. How many people will be called after this continues for 12 terms?

2, 4, 8, 16, 32, 64, 128, 256, 512, 1024

$$a_n = 2(2)^{n-1}$$

$$a_{12} = 2(2)^{11} = 4096 \text{ calls}$$

37) How are linear functions drastically different from exponential functions?

| Linear | Exponential |
| --- | --- |
| ↓ | ↓ |
| constant rate of change | constant <u>percent</u> rate of change |
| line | curve |
| Range: all real #'s | Range: limited by asymptote boundary line |

# Solving Exponential Equations Key

13) $4^{-2x} \cdot 4^{x} = 64$

$4^{-2x+x} = 4^3$

$-2x+x = 3$

$-x = 3$

$\boxed{x = -3}$

14) $6^{-2x} \cdot 6^{-x} = \frac{1}{216}$

$6^{-2x-x} = 6^{-3}$

$-2-x = -3$

$-x = -1$

$\boxed{x = 1}$

15) $2^{x} \cdot \frac{1}{32} = 32$

$2^x \cdot 2^{-5} = 2^{5}$

$2^{x-5} = 2^{5}$

$x-5 = 5$

$\boxed{x = 10}$

16) $2^{-3p} \cdot 2^{2p} = 2^{2p}$

$2^{-3p+2p} = 2^{2p}$

$2^{-p} = 2^{2p}$

$-p = 2p$

$0 = 3p$ $\boxed{p = 0}$

17) $64 \cdot 16^{-3x} = 16^{3x-2}$

$4^3 \cdot 4^{2(-3x)} = 4^{2(3x-2)}$

$4^{3+-6x} = 4^{6x-4}$

$3-6x = 6x-4$

$7 = 12x$

$\boxed{x = \frac{7}{12}}$

18) $\frac{81^{3n+2}}{243^{-n}} = 3^4$

$\frac{3^{4(3n+2)}}{3^{5(-n)}} = 3^4$

$\frac{3^{12n+8}}{3^{-5n}} = 3^4 \rightarrow 3^{12n+8+5n} = 3^4$

$3^{17n+8} = 3^4$

$17n+8 = 4$

$17n = -4$

$\boxed{n = -\frac{4}{17}}$

19) $81 \cdot 9^{-2b-2} = 27$

$3^4 \cdot 3^{-4b-4} = 3^3$

$3^{-4b} = 3^3$

$-4b = 3$

$\boxed{b = -\frac{3}{4}}$

20) $9^{-3x} \cdot 9^{x} = 27$

$3^{-6x} \cdot 3^{2x} = 3^3$

$3^{-4x} = 3^3$

$-4x = 3$

$\boxed{x = -\frac{3}{4}}$

21) $\left(\frac{1}{6}\right)^{3x+2} \cdot 216^{3x} = \frac{1}{216}$

$6^{-1(3x+2)} \cdot 6^{3(3x)} = 6^{-3}$

$6^{-3x-2+9x} = 6^{-3}$

$6x-2 = -3$

$6x = -1$

$\boxed{x = -\frac{1}{6}}$

22) $243^{k+2} \cdot 9^{2k-1} = 9$

$3^{5k+10} \cdot 3^{4k-2} = 3^2$

$3^{9k+8} = 3^2$

$9k+8 = 2$

$9k = -6$

$\boxed{k = -\frac{2}{3}}$

23) $16^{r} \cdot 64^{3-3r} = 64$

$4^{2r} \cdot 4^{9-9r} = 4^3$

$4^{-7r+9} = 4^3$

$-7r+9 = 3$

$-7r = -6$

$\boxed{r = \frac{6}{7}}$

24) $16^{2p-3} \cdot 4^{-2p} = 2^4$

$2^{8p-12} \cdot 2^{-4p} = 2^4$

$2^{4p-12} = 2^4$

$4p-12 = 4$

$4p = 16$

$\boxed{p = 4}$

# Geometric Sequences Worksheet

Ms. Barnaby
Mathematics 12 Advanced

Name: Key

Date: ____________________

1. Find the next 3 terms in the geometric sequences below.

a) {2,6, …} b) {10, 5,…} c) {12, -6,…}

2. Find the 6th term in each of the following geometric sequences.

a) {3, 6, 12, 24,…} b) {2, 10, 50, …} c) {512, 256, 128…}

3. Find the 9th term in each of the following geometric sequences.

a) {1, 3, 9, 27,…} b) {12, 18, 27, …} c) $\{\frac{1}{16}, -\frac{1}{8}, \frac{1}{4}, -\frac{1}{2} \ldots\}$ d) {a, ar, $ar^2$,…}

4. Consider the sequence {5, 10, 20, 40,…}

a) Show that the sequence is geometric. b) Find the equation for the general term.
c) Find the value of the 15th term.

5. Consider the sequence $\{12, -6, 3, -\frac{3}{2}, \ldots\}$

a) Show that the sequence is geometric. b) Find the equation for the general term.
c) Find the value of the 13th term (as a fraction).

~~6. Find k given that the following sequences are geometric.~~

~~a) {7, k, 28…} b) {k, 3k, 20-k,…} c) {k, k+8, 9k…}~~

7. Find the general term of a geometric sequence which has:

a) $u_4 = 27$ and $u_7 = 192$ b) $u_3 = 5$ and $u_7 = \frac{5}{4}$

SOLUTIONS

1. a) 18, 54, 162 b) 2.5, 1.25, 0.625 c) 3, -1.5, 0.75

2. a) 96 b) 6250 c) 16

3. a) 6561 b) $\frac{19683}{64}$ c) 16 d) $ar^8$

4. a) r = 3 b) $u_n = 5(2)^{n-1}$ c) $u_{15} = 81\,920$

5. a) $r = -\frac{1}{2}$ b) $u_n = 12(-\frac{1}{2})^{n-1}$ c) $u_{13} = \frac{3}{1024}$

6. a) $\pm 14$ b) 2 c) -2 or 4

7. a) $u_n = 3(2)^{n-1}$ b) $u_n = 10(\pm\sqrt{2})^{1-n}$

GPS Advanced Algebra Unit 3 Name: Key Pd:_____

# Exponential Growth and Decay Word Problems

1. Find a bank account balance if the account starts with \$100, has an annual rate of 4%, and the money left in the account for 12 years.

$P_t = 100(1+.04)^t$ $P_{12} = 100(1.04)^{12} =$ \$160.10

2. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994?

$P_t = 285(1+.75)^t$ $P_9 = 285(1.75)^9 =$

43,871 subscribers

3. Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If we start with only one bacteria which can double every hour, how many bacteria will we have by the end of one day?

$P_t = 1(2)^t$ $t =$ every hour

$P_{24} = 2^{24} =$ 16,777,216 bacteria

4. Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds?

$P_t = 128(1-\frac{1}{2})^t$

$P_5 = 128(\frac{1}{2})^5 =$ 4 players

5. The population of Winnemucca, Nevada, can be modeled by $P=6191(1.04)^t$ where t is the number of years since 1990. What was the population in 1990? By what percent did the population increase by each year?

6191 people 4%

$$\begin{array}{r} 2011 \\ -1960 \\ \hline 51 \end{array}$$

6. You have inherited land that was purchased for \$30,000 in 1960. The value of the land increased by approximately 5% per year. What is the approximate value of the land in the year 2011?

$P_t = 30{,}000(1+.05)^t$ $P_{51} = 30{,}000(1.05)^{51} =$

\$361,223.09

7. During normal breathing, about 12% of the air in the lungs is replaced after one breath. Write an exponential decay model for the amount of the original air left in the lungs if the initial amount of air in the lungs is 500 mL. How much of the original air is present after 240 breaths?

$P_t = 500(1-.12)^t$ $t =$ one breath

$P_{240} = 500(.88)^{240} = 2.4 \times 10^{-11}$ mL

8. An adult takes 400 mg of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about 29%. How much ibuprofen is left after 6 hours?

$t$ = each hour

$P_t = 400(1-.29)^t$

$P_6 = 400(.71)^6 = \underline{51.24 \text{ mg}}$

9. You deposit $1600 in a bank account. Find the balance after 3 years for each of the following situations:
   a. The account pays 2.5% annual interest compounded monthly.

   $P_3 = 1600\left(1+\frac{.025}{12}\right)^{12(3)} = \underline{\$1724.48}$

   b. The account pays 1.75% annual interest compounded quarterly.

   $P_3 = 1600\left(1+\frac{.0175}{4}\right)^{4(3)} = \underline{\$1686.05}$

   c. The account pays 4% annual interest compounded yearly.

   $P_3 = 1600\left(1+\frac{.04}{1}\right)^{3} = \underline{\$1799.78}$

10. You buy a new computer for $2100. The computer decreases by 50% annually. When will the computer have a value of $600?

$P_t = 2100(1-.5)^t$

$600 = 2100(.5)^t$

$t \approx \underline{1.8 \text{ years}}$

($603)

11. You drink a beverage with 120 mg of caffeine. Each hour, the caffeine in your system decreases by about 12%. How long until you have 10mg of caffeine?

$P_t = 120(1-.12)^t$

$10 = 120(.88)^t$

$t \approx \underline{19.4 \text{ hours}}$ (10.05 mg.)

12. The foundation of your house has about 1,200 termites. The termites grow at a rate of about 2.4% per day. How long until the number of termites doubles?

$P_t = 1200(1+.024)^t$  $t$ = per day

$2400 = 1200(1.024)^t$

$t \approx \underline{30 \text{ days}}$

29 days: 2387

30 days: 2444